

Mock Test -1

Senior Secondary Examination, 2023

Mathematics

Time : 3

Max. Marks : 80

General Instructions to the Examinees:

- (1) Candidate must write first his/her Roll No. on the question paper compulsorily.
- (2) All the questions are compulsory.
- (3) Write the answer to each question in the given answer book only.
- (4) For question having more than one part the answers to those part are to be written together in continuity.
- (5) Write down the serial number of the question before attempting it.

Section - A

1. $\int \sqrt{4-x^2} dx = ?$

(1) None of these

(2) $\frac{x}{2}\sqrt{4-x^2} + 2\sin^{-1}\frac{x}{2} + C$

(3) $x\sqrt{4-x^2} + \sin^{-1}\frac{x}{2} + C$

(4) $\frac{1}{2}x\sqrt{4-x^2} - 2\sin^{-1}\frac{x}{2} + C$

2. $[\hat{i} \hat{j} \hat{k}] = ?$

(1) 3

(2) 1

(3) 2

(4) 0

3. If $P(A) = \frac{2}{5}$, $P(B) = \frac{3}{10}$ and $P(A \cap B) = \frac{1}{5}$, then

$P\left(\frac{A'}{B'}\right) \cdot P\left(\frac{B'}{A'}\right)$ is equal to

(1) $\frac{25}{42}$

(2) $\frac{5}{6}$

(3) $\frac{5}{7}$

(4) 1

4. The angle between two lines having direction ratios 1, 2, 2 and $(\sqrt{3}-1), (-\sqrt{3}-1), 4$ is

(1) $\frac{\pi}{4}$

(2) $\frac{\pi}{3}$

(3) $\frac{\pi}{6}$

(4) $\frac{\pi}{2}$

5. Two numbers are selected and random from integers 1 through 9. If the sum is even, what is the probability that both numbers are odd ?
- (1) $\frac{5}{8}$ (2) $\frac{1}{6}$
(3) $\frac{4}{9}$ (4) $\frac{2}{3}$
6. $\int e^x \left(\frac{1-x}{1+x^2} \right)^2 dx$ is equal to
- (1) $\frac{-e^x}{1+x^2} + C$ (2) $\frac{e^x}{(1+x^2)} + C$
(3) $\frac{-e^x}{(1+x^2)^2} + C$ (4) $\left(\frac{e^x}{(1+x^2)^2} \right) + C$
7. The direction ratios of two lines are 3, 2, -6 and respectively. The acute angle between these lines is
- (1) $\cos^{-1} \left(\frac{5}{18} \right)$ (2) $\cos^{-1} \left(\frac{8}{21} \right)$
(3) $\cos^{-1} \left(\frac{5}{21} \right)$ (4) $\cos^{-1} \left(\frac{3}{20} \right)$
8. The area of the region bounded by the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$ is
- (1) $20\pi^2$ sq. units
(2) 25π sq. units
(3) 20π sq. units
(4) $16\pi^2$ sq. units
9. If \vec{a} and \vec{b} are unit vectors inclined at an angle θ , then the value of $|\vec{a} - \vec{b}|$ is
- (1) $2\cos \frac{\theta}{2}$ (2) $2\sin \frac{\theta}{2}$
(3) $2\cos \theta$ (4) $2\sin \theta$
10. The area enclosed by the circle $x^2 + y^2 = 2$ is equal to
- (1) $4\pi^2$ sq. units (2) 4π sq. units
(3) 2π sq. units (4) $2\sqrt{2}\pi$ sq. units
11. The degree of the differential equation $\left[1 + \left(\frac{dy}{dx} \right)^2 \right]^{\frac{3}{2}} = \frac{d^2y}{dx^2}$ is
- (1) 2 (2) $\frac{3}{2}$
(3) not defined (4) 4
12. $\int \sin^3(2x+1) dx = ?$
- (1) $\frac{1}{2} \cos(2x+1) + \frac{1}{3} \cos^3(2x+1) + C$
(2) $-\frac{1}{2} \cos(2x+1) + \frac{1}{6} \cos^3(2x+1) + C$
(3) $\frac{1}{8} \sin^4(2x+1) + C$
(4) None of these
13. If $\begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix}$ and $A + A' = 1$, If the value of α is
- (1) $\frac{\pi}{6}$ (2) $\frac{\pi}{3}$
(3) $\frac{3\pi}{2}$ (4) π
14. Function $f(x) = 2x^3 - 9x^2 + 12x + 29$ is monotonically decreasing when
- (1) $x > 2$ (2) $1 < x < 2$
(3) $x < 2$ (4) $x > 3$
15. Let A be a non-singular square matrix of order 3×3 . Then $|\text{adj } A|$ is equal to
- (1) $|A|$ (2) $3|A|$
(3) $|A|^3$ (4) $|A|^2$
16. If A is a 3×3 matrix such that $|A| = 8$, then $|3A|$ equals.
- (1) 8 (2) 72
(3) 216 (4) 24
17. What is the equation of a curve passing through (0, 1) and whose differential equation is given by $dy = y \tan x dx$?
- (1) $y = \sec x$ (2) $y = \sin x$
(3) $y = \text{cosec } x$ (4) $y = \cos x$

18. Domain of $\cos^{-1}x$ is

- (1) $[-1, 0]$ (2) $[0, 1]$
(3) None of these (4) $[-1, 1]$

19. **Assertion (A)** : If manufacturer can sell x items at a

price of Rs. $\left(5 - \frac{x}{100}\right)$ each. The cost price of x items

is Rs. $\left(\frac{x}{5} + 100\right)$. Then, the number of items he should sell to earn maximum profit is 240 items.

Reason (R) : The profit for selling x items is given by

$$\frac{24}{5}x - \frac{x^2}{100} - 300.$$

- (1) Both A and R are true and R is the correct explanation of A
(2) Both A and R are true but R is not the correct explanation of A.
(3) A is true but R is false.
(4) A is false but R is true.

20. **Assertion (A)** : The matrix $A = \begin{bmatrix} 1 & 2 \\ 4 & 8 \end{bmatrix}$ is singular.

Reason (R) : A square matrix A is said to be singular, if $|A| = 0$.

- (1) Both A and R are true and R is the correct explanation of A
(2) Both A and R are true but R is not the correct explanation of A.
(3) A is true but R is false.
(4) A is false but R is true.

Section - B

21. Write the cofactor a_{12} in the matrix $\begin{bmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{bmatrix}$

or

Solve the system of equations by matrix method

$$8x + 4y + 3z = 18$$

$$2x + y + z = 5$$

$$x + 2y + z = 5$$

22. Find the value of

$$\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right) + \cos^{-1}\left(\frac{1}{\sqrt{3}}\right) + \tan^{-1}\left[\sin\left(\frac{-\pi}{2}\right)\right]$$

23. Verify that $y = (a + bx)e^{2x}$ is the general solution of

$$\text{the differential equation } \frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 0$$

24. Given the probability that A can solve a problem is $\frac{2}{3}$, and the probability that B can solve the same problem is $\frac{3}{5}$, find the probability that at least one of A and B will solve the problem.

25. For what value of λ are the vectors \vec{a} and \vec{b} perpendicular to each other? where;

$$\vec{a} = \lambda\hat{i} + 2\hat{j} + \hat{k} \text{ and } \vec{b} = 4\hat{i} - 9\hat{j} + 2\hat{k}$$

Section - C

26. Find the particular solution of the differential equations

$$e^x \sqrt{1-y^2} dx + \frac{y}{x} dy = 0, \text{ given that } y = 1, \text{ when } x = 0.$$

OR

Solve the initial value problem :

$$(x^2 + 1)y' - 2xy = (x^4 + 2x^2 + 1) \cos x, y(0) = 0$$

27. Prove $\int_0^{\frac{\pi}{4}} 2 \tan^3 x dx = 1 - \log 2$

28. Evaluate : $\int \frac{x^2}{(x^4 - x^2 - 12)} dx$

OR

$$\text{Evaluate : } \int \sin^{-1} \sqrt{\frac{x}{a+x}} dx$$

29. Show that the point A (1, 2, 7), B(2, 6, 3) and C(3, 10, -1) are collinear.

OR

Prove using vectors; The quadrilateral obtained by joining midpoint of adjacent side of a rectangle is a rhombus.

30. If $x = a(1 - \cos^3 \theta)$, $y = a \sin^3 \theta$, prove that $\frac{d^2y}{dx^2} = \frac{32}{27a}$

$$\text{at } \theta = \frac{\pi}{6}.$$

31. Sketch the region bounded by the curve $y = 2x - x^2$ and the x-axis and find its area.

Section - D

32. Minimize $Z = x + 2y$ subject to $2x + y \geq 3$, $x + 2y \geq 6$, $x, y \geq 0$. Show that the minimum of Z occurs at more than two points.
33. Show that the function $f: R_0 \rightarrow R_0$, defined as $f(x) = \frac{1}{x}$, is one-one, where R_0 is the set non-zero real number. Is the result true, if the domain R_0 is replaced by N with co-domain being same as R_0 ?

OR

Show that the function $f: R \rightarrow R$ defined by $f(x)$

$$\frac{x}{x^2 + 1}, \forall x \in R, \text{ is neither one-one nor onto.}$$

34. Find the vector equation of the line passing through $(1, 2, 3)$ and parallel to of the planes $\vec{r} \cdot (\hat{i} - \hat{j} + 2\hat{k}) = 5$ and $\vec{r} \cdot (3\hat{i} + \hat{j} + \hat{k}) = 6$. Also find the point of intersection of the line thus obtained with the plane $\vec{r} \cdot (2\hat{i} + \hat{j} + \hat{k}) = 4$

OR

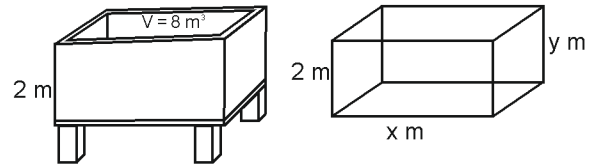
Show that the line $\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 4\hat{k})$

and $\vec{r} = (4\hat{i} + \hat{j}) + \mu(5\hat{i} + 2\hat{j} + \hat{k})$ intersect. Also, find their point intersection.

35. Find $\frac{dy}{dx}$ of the function $x^y + y^x = 1$

Section - E

36. **Read the text carefully and answer the questions:**
One the request of villagers, a construction agency designs a tank with the help of an architect. Tank consist of a rectangular base with rectangular side, open at the top so that its depth is 2 m and volume is $8m^3$ as shown below. The construction of the tank cost Rs. 70 per sq. meter for the base and Rs.45 per square meter for sides.



- (i) Express making cost C in terms of length of rectangle base.
- (ii) If x and y represent the length and breadth of its rectangular base, then find the relation between the variables.
- (iii) Find the value of x of that the cost of construction is minimum.

OR

Verify by second derivative test that cost is minimum at a critical point.

37. **Read the text carefully and answer the questions:**

Three car dealers, say A, B and C deals in three types of cars, namely Hatchback cars, Sedan car, SUV cars. The sales figure of 2019 and 2020 showed that dealer A sold 120 Hatchback, 50 Sedan, 10 SUV cars in 2019 and 300 Hatchback, 150 Sedan, 20 SUV cars in 2020; dealer B sold 100 Hatchback, 30 Sedan, 5 SUV cars in 2019 and 200 Hatchback, 50 Sedan, 6 SUV cars in 2020; dealer C sold 90 Hatchback, 40 Sedan, 2 SUV cars in 2019 and 100 Hatchback, 60 Sedan, 5 SUV cars in 2020.



- (i) Write the matrix summarizing sales data of 2019 and 2020.
- (ii) Find the matrix summarizing sales data of 2020.
- (iii) Find the total numbers of cars sold in two given years, by each dealer?

OR

If each dealer receives a profit of Rs. 50000 on sale of a Hatchback, Rs. 100000 on sale of a Sedan and Rs. 200000 on sale of an SUV, then find the amount of profit received in the year 2020 by each dealer.

38. **Read the text carefully and answer the questions:**
To teach the application of probability a maths teacher arranged a surprise game for 5 of his students namely Govind, Girish, Vinod, abhishek and Ankit. He took a bowl containing tickets numbered 1 to 50

and told the students go one by one and draw two tickets simultaneously from the bowl and replace it after noting the numbers.



- (i) Teacher as Govind, what is the probability that tickets are drawn by Abhishek, shows a prime number on one ticket and a multiple of 4 on other ticket?
- (ii) Teacher ask Girish, what is the probability that tickets drawn by Ankit, shows an even number on first ticket and an odd number on second ticket?